fourth GRADE

Whole Number Operations
<u>Math in Focus</u>

Unit 1 Curriculum Guide:
September 10 – November 7th, 2018

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Unit Overview

Unit 1: Chapters 1-3
In this Unit Students will:
Chapter 1: Place Value of Whole Numbers: Chapter 1 is a critical chapter. The chapter focuses on quantities to 100,000, but CCSS expects understanding to 1,000,000, which can be added. Note that students are expected to be proficient renaming numbers (1200 is 12 hundreds or 120 tens). Students use place value chips instead of base ten proportional materials. These are more abstract. Expanded notation and comparing are emphasized with the place value models and charts. **Chapter 2: Estimation and Number Theory:** In Chapter 2, students address CCSS for estimation and for finding factor pairs as well as recognizing prime and composite numbers. If students are still weak on multiplication facts, use simpler products to get at these ideas. While LCM is not essential yet, this can be introduced at this point even if not all students master it. Estimation should be practiced all year. Front end estimation is introduced as just another way to estimate, that can be useful when numbers are large. If you have trouble with this, skip it. **Chapter 3: Whole Number Multiplication and Division:** Emphasis for Chapter 3 is on building from concrete and pictorial and connecting this to how we notate it symbolically. Students should be working a lot with base ten blocks, place value chips and place value charts. Make sure you don't move too quickly. Two good problems in a class period, discussed at length, are more effective than ten problems done quickly. While 3.1 has only two examples at the concrete level, please include more. Similarly with division in 3.3. If 20 days are not enough, continue to work on this essential chapter
Essential Questions
How are greater numbers read and written? How can whole numbers be compared and ordered? How do the digits in a multi-digit number relate to each other? How can place value help you compare whole numbers? How do you compare numbers? How do you round numbers? What are standard procedures for adding and subtracting whole numbers? How can you use mental math to add and subtract? How can you estimate sums and differences of whole numbers? How do you subtract across zeroes? How can a bar diagram help you solve an addition or subtraction problem? How does place value and properties of operations help in developing and understanding strategies for multiplication and division? How can patterns and properties be used to find some multiplication facts? How can unknown multiplication facts be found by breaking them into know facts? How can unknown division facts be found by thinking about a related multiplication fact? How can we use arrays to understand multiplication? How can we use arrays to find a product? How can you use models to help you record multiplication?

How can you use compatible numbers to estimate products? How can we use our understanding of factors and multiples to help solve multiplication and division problems? How are additive and multiplicative comparisons alike and different? How is using estimation helpful in seeing the reasonableness of an answer when solving problems? How does a story context affect interpreting and using a remainder? How can you use place value and patterns to help you divide mentally? What place- value patterns can be seen when you multiply 1-digit numbers by multiples of 10 and 100? How can you use rounding to estimate when you multiply?
Enduring Understandings
• We can use various strategies to solve word problems • There are patterns between multiplication & division

MIF Chapter 1-3 & Eureka Math Module 1 (TOPIC E,F)			
Activity	**CCSS**	**Estimated Time (# of block)**	**Lesson Notes**
1.1 Numbers to 100,000 (Extra block to focus on manipulative use w/ foam chips and power of ten rule)	4.NBT.1-2, 4.OA.5	2 Blocks	You may wish to assign groups of students to become experts at the different forms taught throughout this lesson: Standard Form Experts, Word Form Experts, and Expanded Form Experts. (Extra block to focus on manipulative use w/ foam chips and power of ten rule)
1.2. Comparing Numbers to 100,000 Day 1 – Compare Day 2 - Order Day 3 – Pattern/Rule	4.NBT.1-2, 4.OA.5	3 Blocks	Guide students to develop a list of steps they can follow when finding the rule for a number pattern. Display the steps for students to use throughout this lesson.
1.3 Adding and Subtracting Multi-Digit Numbers	4.NBT.4	2 Blocks	Students find the place-value chips engaging. Allow them to model as many problems in this lesson a time and resources allow. This tactile experience solidifies the concept of regrouping in a concrete way.
Authentic Assessment #1	3.NBT.2	1/2 Block	
Chapter Test/Performance Task	4.NBT.1-3	1/2 Block	
2.1 Day 1 Use Rounding to Check Reasonableness of Sums and Differences	4.OA.3-4, 4.NBT.2, 4.NBT.5	1/2 Block	Introduction after Pre-test. Focus on use of manipulatives
2.1 Day 2 Use Rounding to Check the Reasonableness of Products	4.OA.3-4, 4.NBT.2, 4.NBT.5	1 Blocks	To check understanding, ask half the students to use rounding to estimate the answers, and the other half to use front-end estimation. Have students compare their results. Discuss which method gave estimates closer to the actual answers and why.
2.1 Day 3 Decide Whether to Find an Estimate or an Exact Answer	4.OA.3-4, 4.NBT.2, 4.NBT.5	1 Block	
Authentic Assessment #2	4.NBT.4	1/2 Block	
2.2 Day 1 Intro. Factors	4.OA.4	1 Block	Throughout this lesson, periodically review basic multiplication and division facts with students to help them become more proficient in identifying common factors.
2.2 Day 1 Break Down Whole Numbers into Factors	4.OA.4	1 Block	When listing the factors of a number students may forget to list 1 and the number itself. It is critical that they list these factors, so that they will understand

			the upcoming lesson on prime numbers. *Can use Factor Rainbow Project Idea to assist students with understanding
2.2 Day 2 Identify Prime and Composite Numbers	4.OA.4	1 Block	Some students may name only factors with which they are familiar. For example, they may think the only factors of 42 are 6 and 7. Encourage students to divide greater number by 2, 3, and 4 to find additional factors.
2.3 Find Multiples of a Number	4.OA.4	1 Block	Throughout this lesson, periodically review basic multiplication facts with students to help them become proficient in identifying common multiples.
2.3 Find Common Multiples of Two Whole Numbers	4.OA.4, 4.NBT.2	1 Block	Use Multiple project idea.
2.4 Day 1 Multiplying a 2-Digit Number by a 1-Digit Number Using an Array Model	4.OA.5	2 Block	Have students compare array and area models. ***Ask:*** Which one is easier? Which one works better when the 2-digit number is large? Tell students that an advantage of an area model is that it can be used when working with fractions. The array model cannot.
Chapter Test/Performance Task	4.OA.3-4, 4.NBT.2, 4.NBT.5	2 Block	
3.1 Day 1 Represent Multiplication as Repeated Addition, a Rectangular Array, and the Area of a Rectangle	4.NBT.1, 4.NBT.5, 4.OA.2	1 Block	
3.1 Day 2 Multiplying Using the Place Value of Each Digit	4.NBT.1, 4.NBT.5, 4.OA.2	1 Block	Students may forget to add the regrouped numbers when they multiply each place. Have students circle the regrouped number, and then cross it out after they have added it.
3.2 Day 1 Multiply by Tens	4.NBT.3, 4.NBT.5	1 Block	After teaching each Learn section, have students work in pairs to write a question each, based on what they have learned. Then have pairs answer each other's questions.
3.2 Day 3 Use a Number line to Estimate Products	4.NBT.3, 4.NBT.5	1 Block	
3.3 Day 1 Modeling Division With Regrouping in Hundreds, Tens, and Ones Optional – Display both traditional and partial quotient	4.NBT.1, 4.NBT.6	2 Blocks	You may wish to have students work in groups using base-ten blocks to act out this first Learn activity. Encourage students to explain to each other how and when to regroup as they model each step.
3.4 Day 1 Divide With No Remainder	4.NBT.6	2 Block	
3.4 Day 2 Find the Quotient and	4.NBT.6	1 Block	Point out that when writing the entire

the Remainder			quotient the remainder is written to the right of the dividend with an R preceding it. Show students how this is done before they attempt Guided Learning Exercise 11 and 12.
3.4 Day 3 Estimate Quotients Using Related Multiplication Facts	4.NBT.6	2 Block	Some students may ignore zeros in the dividend when dividing. Explain that zeros are like any other digit in the dividend and should always have a digit written above them when finding the quotient.
Authentic Assessment #3	4.NBT.6	1/2 Block	
3.5 Day 1 Solve 3-Step Problems Using Models	4.OA.1-3	1 Block	Before solving each problem, check that students know what question they need to answer to solve the problem. To maintain focus on obtaining the solution, have students write the problem at the top of their paper and check that each solution step leads to answering the specific question.
3.5 Day 2 Solve 3-Step Problems Using Models	4.OA.1-3	1 Block	
Review - Problem Solving	4.OA.2-3	1 Block	
Chapter Test/Performance Task	4.NBT.5-6, 4.OA.3	1 Block	
Authentic Assessment #4	4.NBT.1, 4.OA.1, 4.NBT.5	1 Block	
Eureka Math Module 1: Algorithms For Addition/Subtraction (TOPIC E,F)			

Topic E: Multi-Digit Whole Number Subtraction	Lesson 13	Use place value understanding to decompose to smaller units once using the standard subtraction algorithm and apply the algorithm to solve word problems using tape diagrams. https://www.youtube.com/watch?v
	Lesson 14	Use place value understanding to decompose to smaller units up to 3 times using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams. https://www.youtube.com/watch?v

	Lesson 15	Use place value understanding to fluently decompose to smaller units multiple times in any place using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams. https://www.youtube.com/watch?v
	Lesson 16	Solve two-step word problems using the standard subtraction algorithm fluently modeled with tape diagrams and assess the reasonableness of answers using rounding. https://www.youtube.com/watch?v
Topic F: Addition and Subtraction Word Problems	Lesson 18	Solve multi-step word problems modeled with tape diagrams and assess the reasonableness of answers using rounding. https://www.youtube.com/watch?v
	Lesson 19	Create and solve multi-step word problems from given tape diagrams and equations. https://www.youtube.com/watch?v

Common Core State Standards

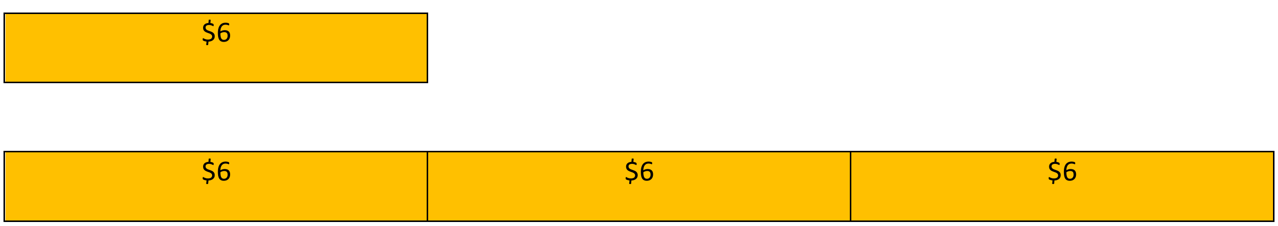
4.OA.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. A *multiplicative comparison* is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "*a* is *n* times as much as *b*").

Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.
Students should be given opportunities to write and identify equations and statements for multiplicative comparisons.
Example:
5 x 8 = 40.
Sally is five years old. Her mom is eight times older. How old is Sally's Mom?
5 x 5 = 25

- Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?

4.OA.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems. In a multiplicative comparison, the underling question is *what amount would be added to one quantity* in order to result in the other. In a multiplicative comparison, the underlying question is *what factor would multiply one quantity* in order to result in the other.



3 x *B*=*R*

3 x $6 = $18

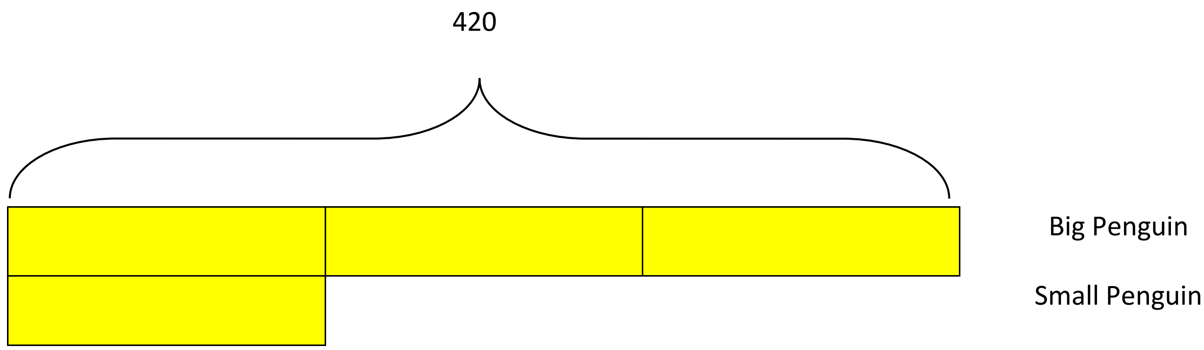
Tape diagram used to solve the Compare problem in Table 3

B is the cost of a blue hat in dollars

R is the cost of a red hat in dollars

A tape diagram used to solve a Compare problem

A big penguin will eat 3 times as much fish as a small penguin. The big penguin will eat 420 grams of fish.



B=number of grams the big penguin eats

S=number of grams the small penguin eats

3 x S=B

3 x S=420

S=140

S + B=140 + 420

=560

4.OA.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies,including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.

Example 1:

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total? Some typical estimation strategies for this problem:

Student 1	**Student 2**	**Student 3**
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to: • front-end estimation with adjusting (using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts), • clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate), • rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values), • using friendly or compatible numbers such as factors (students seek to fit numbers together - e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000), • using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).	
4.OA.4	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8. A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number. <u>Prime vs. Composite</u>: A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors. Students investigate whether numbers are prime or composite by • building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number) • finding factors of the number Students should understand the process of finding factor pairs so they can do this for any number 1 - 100, <u>Example</u>: Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12. Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20). To determine if a number between1-100 is a multiple of a given one-digit number, some helpful hints include the following: • all even numbers are multiples of 2 • all even numbers that can be halved twice (with a whole number result) are multiples of 4 • all numbers ending in 0 or 5 are multiples of 5	

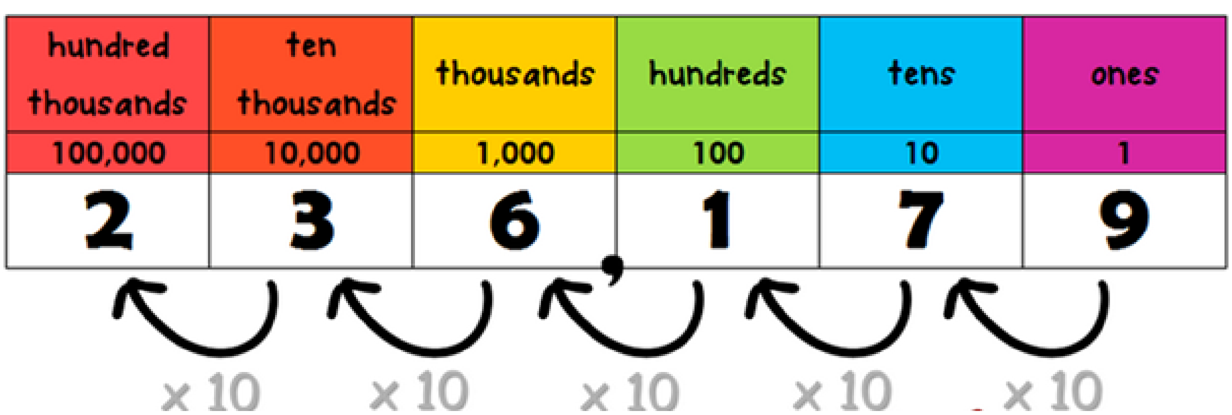
4.OA.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3"and the starting number 1, generateterms in the resulting sequence and observe that the terms appear to alternate between odd and evennumbers. Explain informally why the numbers will continue to alternate in this way.*

Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations.
Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

<u>Example</u>:
Pattern Rule Feature(s)
3, 8, 13, 18, 23, 28, … Start with 3, add 5 The numbers alternately end with a 3 or 8
5, 10, 15, 20 …
Start with 5, add 5 The numbers are multiples of 5 and end with either 0 or 5. The numbers that end with 5 are products of 5 and an odd number.
The numbers that end in 0 are products of 5 and an even number.
After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.

4.NBT.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 =10 by applying concepts of place valueand division.*

This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with.
In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. Because of this, multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left.



4.NBT.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons

This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is 285 = 200 + 80 + 5. Written form or number name is two hundred eighty-five.

However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones.
To read numerals between 1,000 and 1,000,000, students need to understand the role of commas.
Students should also be able to compare two multi-digit whole numbers using appropriate symbols.

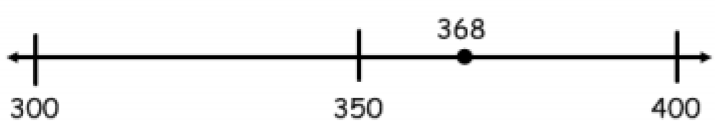
4.NBT.3	Use place value understanding to round multi-digit whole numbers to any place.

This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding.
The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

Example:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1	**Student 2**
First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40+20=60. 300- 60 = 240, so we need about 240 more bottles.

Example:
Round 368 to the nearest hundred.
This will either be 300 or 400, since those are the two hundreds before and after 368.
Draw a number line, subdivide it as much as necessary, and determine whether 368 is closer to 300 or 400.
Since 368 is closer to 400, this number should be rounded to 400



4.NBT.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract.

This standard refers to fluency, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using a variety strategies such as the distributive property). This is the first grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use.

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. With this in mind, minor variations in methods of recording standard algorithms are acceptable.
As with addition and subtraction, students should use methods they understand and can explain. Visual representations such as area and array diagrams that students draw and connect to equations and other written numerical work are useful for this purpose. By reasoning repeatedly about the connection between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities.

Computation of 8x549 connected to an area model

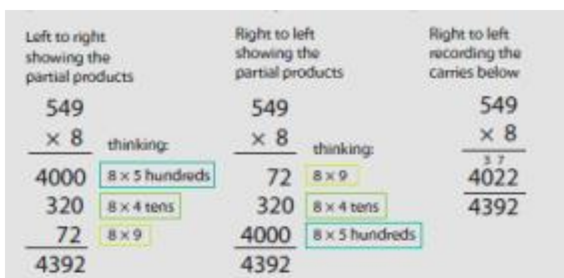
	549=500	+40	+9
8	8 X 500= 8 X 5 hundreds= 40 hundreds	8 X 40 8 X 4 tens= 32 tens	8 X 9 =72

Each part of the region above corresponds t one of the terms in the computation below.

8x549=8x(500+40+9)
=8x500+8x40+8x9

This can also be viewed as finding how many objects are in 8 groups of 549 objects by finding the cardinalities of 8 groups of 500, 8 groups of 40, and 8 groups of 9, then adding them.

Computation of 8x549: Ways to record general methods



The first method proceeds from left to right, and the others from right to left. In the third method, the digits representing new units are written below the line rather than above 549, thus keeping the digits of the products close to each other, e.g., the 7 from 8x9=72 is written diagonally to the left rather than above the 4 in 549.

4.NBT.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication is an expectation in the 5th grade.

Another part of understanding general base-ten methods for multi-digit multiplication is understanding the role played by the distributive property. This allows numbers to be decomposed into base-ten units, products of the units to be computed, and then combined. By decomposing the factors into like base-ten units and applying the distributive property, multiplication computations are reduced to single-digit multiplications and products of numbers with multiples of 10, of 100, and of 1000. Students can connect diagrams of areas or arrays to numerical work to develop understanding of general base-ten multiplication methods. Computing products of two two-digit numbers requires using the distributive property several times when the factors are decomposed into base-ten units.

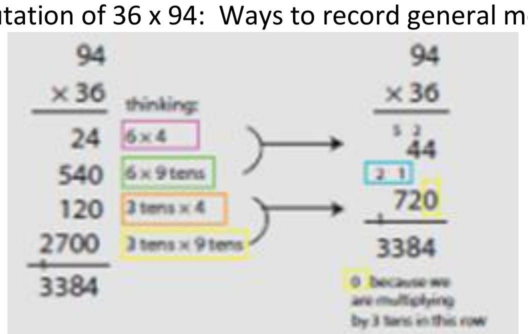
<u>Example</u>: 36 X 94=(30 + 6) X (90+4)
=(30+6) X 90(30 +6) X 4
= 30 X90 + 6 X 90 + 30 X 4 + 6 X 4

Computation of 36 x 94 connected with an area model.

	90	+4
30	30 X 90= 3 tens 9 tens= 27 hundreds= 2,700	30 X 4= 3 tens X 4= 12 tens 120
6	6 X 90= 6 X 9 tens= 54 tens= 540	6 X 4=24

The products of like base-ten units are shown as parts of a rectangular region.

Computation of 36 x 94: Ways to record general methods



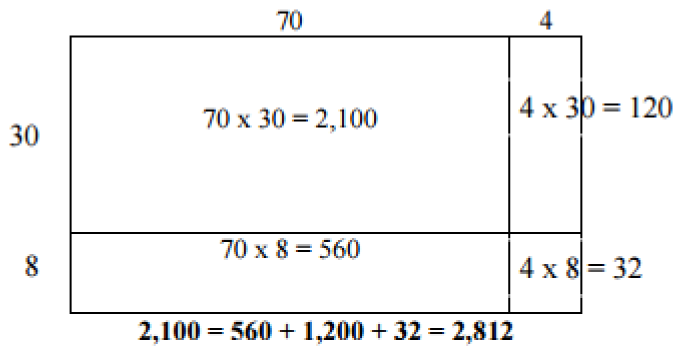
These proceed from right to left, but could go left to right. On the right, digits that represent newly composed tens and hundreds are written below the line instead of above 94. The digits 2 and 1 are surrounded by a blue box. The 1 from 30 x 4 -120 is placed correctly in the hundreds place and the digit 2 from 30 x 90= 2,700 is placed correctly in the thousands place. If these digits had been place above 94, they would be in incorrect places. Note that the 0(surrounded by a yellow box)in the ones place of the second line of the method on the right is there because the whole line of digits is produced by multiplying by 30 (not 3).

This standard calls for students to multiply numbers using a variety of strategies.
<u>Example</u>:
There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?

<u>Student 1</u>	**<u>Student 2</u>**	**<u>Student 3</u>**

25 x12 I broke 12 up into 10 and 2 25 x 10 = 250 25 x 2 = 50 250 +50 = 300	25 x 12 I broke 25 up into 5 groups of 5 5 x 12 = 60 I have 5 groups of 5 in 25 60 x 5 = 300	25 x 12 I doubled 25 and cut 12 in half to get 50 x 6 50 x 6 = 300

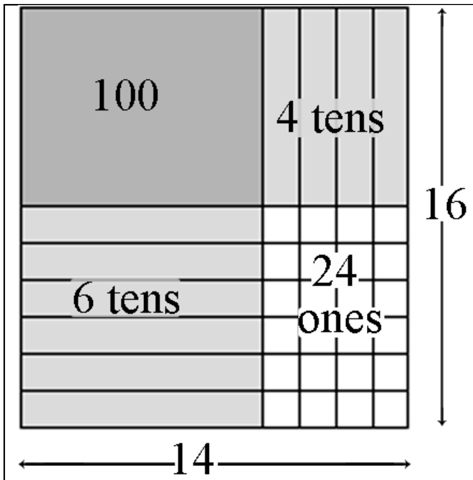
<u>Example</u>:
What would an array area model of 74 x 38 look like?



8 x 4 =32
30 x4=120
70 x 30 = 2,100
70 x 8 = 560
2,100+ 560 + 120 + 32 = 2,812

<u>Example</u>:
To illustrate 154 x 6 students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property, 154 x 6 = (100 + 50 + 4) x 6 = (100 x 6) + (50 X 6) + (4 X 6) =
600 + 300 + 24 = 924.

The area model below shows the partial products.
14 x 16 = 224



$100 + 40 + 60 + 24 = 224$

Using the area model, students first verbalize
their understanding:

• 10 x 10 is 100
• 4 x 10 is 40
• 10 x 6 is 60, and
• 4 x 6 is 24.

They use different strategies to record this type of thinking.
Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.
25x24
400 (20 x 20)
100 (20 x 5)
80 (4 x 20)
20 (4 x 5)
600

4.NBT.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

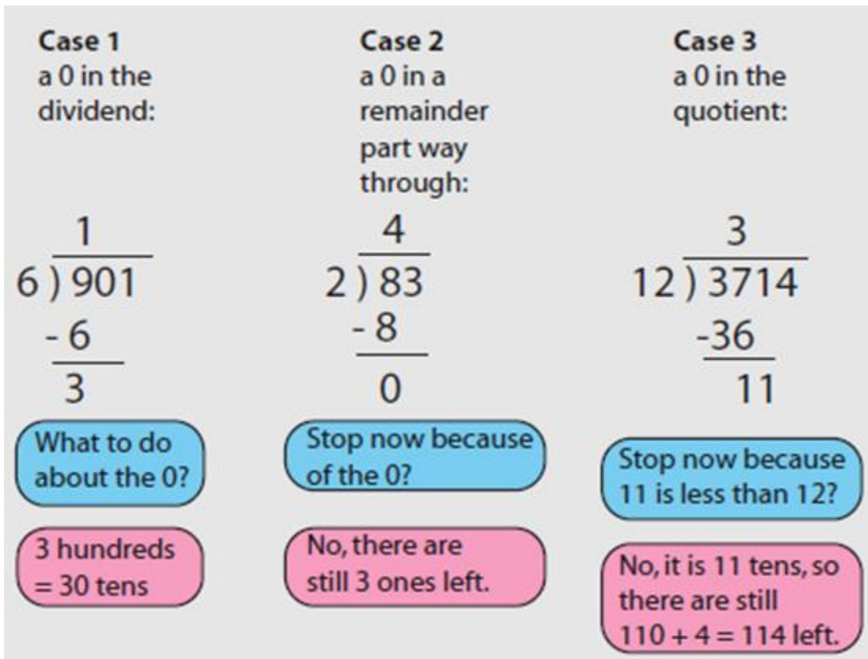
In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

General methods for computing quotients of multi-digit numbers and one-digit numbers rely on the same understandings as for multiplication, but cast in terms of division. One component is quotients of multiples of 10, 100, or 1000 and one-digit numbers. For example, 42 ÷ 6 is related to 420 ÷ 6 and 4200 ÷ 6. Students can draw on their work with multiplication and they can also reason that 4200 6 means partitioning 42 hundreds into 6 equal groups, so there are 7 hundreds in each group.

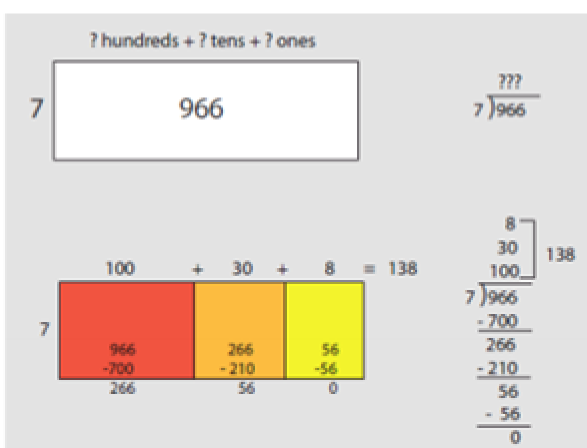
Another component of understanding general methods for multi-digit division computation is the idea of decomposing the dividend into like base-ten units and finding the quotient unit by unit, starting with the largest unit and continuing on to smaller units. As with multiplication, this relies on the distributive property. This can be viewed as finding the side length of a rectangle (the divisor is the length of the other side) or as allocating objects (the divisor is the number of groups).
Multi-digit division requires working with remainders. In preparation for working with remainders, students

can compute sums of a product and a number, such as 4 x 8 + 3. In multi-digit division, students will need to find the greatest multiple less than a given number. For example, when dividing by 6, the greatest multiple of 6 less than 50 is 6 x 8 = 48. Students can think of these “greatest multiples” in terms of putting objects into groups. For example, when 50 objects are shared among 6 groups, the largest whole number of objects that can be put in each group is 8, and 2 objects are left over. (Or when 50 objects are allocated into groups of 6, the largest whole number of groups that can be made is 8, and 2 objects are left over.) The equation 6 x 8 + 2 = 50 (or 8 x 6 + 2 = 50) corresponds with this situation.
Cases involving 0 in division may require special attention.

Cases Involving 0 In Division

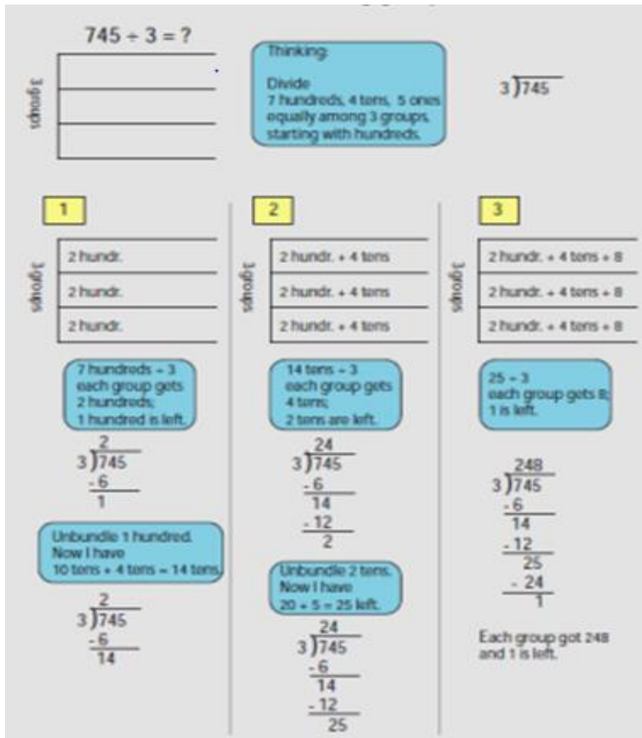


Division as finding side length.



966/7 is viewed as finding the unknown side length of a rectangular region with an area of 966 square units and a side of the length 7 units. The amount of hundreds is found, the tens, then ones. This yields a decomposition into three regions of dimensions 7 by 100, 7 by 30, ad 7 by 8. It can be connected with the decomposition of 966 as 7 x 100 +7 x 30+7 x8. By the distributive property, this is 7 x(100+30+8), so the unknown side length is 138. In the recording on the right, amounts of hundreds,, tens and ones are represented by numbers rather tha by digits, e.g. 700 instead of 7.

Division As Finding Group Size



745/3 can be viewed as allocating 745 objects bundled in 7 hundreds, 4 tens, and 3 ones equally among 3 groups. In step 1, the 2 indicates that each group got 2 hundreds, the 6 is the number of hundreds allocated. After Step 1, the remaining hundred is decomposed as 10 tens and combined with the 4 tens (in 745) to make 14 tens.

Example:
A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?
• Using Base 10 Blocks: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.
• Using Place Value: 260 ÷ 4 = (200 ÷ 4) + (60 ÷ 4)
• Using Multiplication: 4 x 50 = 200, 4 x 10 = 40, 4 x 5 = 20; 50 + 10 + 5 = 65; so 260 ÷ 4 = 65
This standard calls for students to explore division through various strategies.

Example:
There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams get created?
Example:

Using an Open Array or Area Model

After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5th grade.

Example: 150 ÷ 6

Student 1	Student 2	Student 3
592 divided by 8 There are 70 8's in 560 592 - 560 = 32 There are 4 8's in 32 70 + 4 = 74	592-400 50 192-160 20 32-32 4 0 592 divided by 8 I know that 10 8's is 80 If I take out 50 8's that is 400 592 - 400 = 192 I can take out 20 more 8's which is 160 192 - 160 = 32 8 goes into 32 4 times I have none left I took out 50, then 20 more, then 4 more That's 74	I want to get to 592 8 x 25 = 200 8 x 25 = 200 8 x 25 = 200 200 + 200 + 200 = 600 600 - 8 = 592 I had 75 groups of 8 and took one away, so there are 74 teams

M : Major ContentS: Supporting ContentA : Additional Content

Common multiplication and division situations. [1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	**3 x ? = 18**, and **18 ÷ 3 = ?**	**? x 6 = 18**, and **18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS [2], AREA [3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	**a x b = ?**	**a x ? = p** and **p ÷ a = ?**	**? x b = p**, and **p ÷ b = ?**

[1] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

MIF Lesson Structure

	LESSON STRUCTURE	RESOURCES	COMMENTS
PRE TEST	**Chapter Opener** Assessing Prior Knowledge *The Pre Test serves as a diagnostic test of readiness of the upcoming chapter*	**Teacher Materials** Quick Check Pretest (Assessm't Bk) Recall Prior Knowledge **Student Materials** Student Book (Quick Check); Copy of the Pre Test; Recall prior Knowledge	Recall Prior Knowledge (RPK) can take place just before the pre-tests are given and can take 1-2 days to front load prerequisite understanding Quick Check can be done in concert with the RPK and used to repair student misunderstandings and vocabulary prior to the pre-test ; Students write Quick Check answers on a separate sheet of paper Quick Check and the Pre Test can be done in the same block *(See Anecdotal Checklist; Transition Guide)* Recall Prior Knowledge – Quick Check – Pre Test
DIRECT ENGAGEMENT	**Direct Involvement/Engagement** Teach/Learn *Students are directly involved in making sense, themselves, of the concepts – by interacting the tools, manipulatives, each other, and the questions*	**Teacher Edition** 5-minute warm up Teach; Anchor Task **Technology** Digi **Other** Fluency Practice	• The Warm Up activates prior knowledge for each new lesson • Student Books are CLOSED; Big Book is used in Gr. K • Teacher led; Whole group • Students use concrete manipulatives to explore concepts • A few select parts of the task are explicitly shown, but the majority is addressed through the hands-on, constructivist approach and questioning • Teacher facilitates; Students find the solution
GUIDED LEARNING	**Guided Learning and Practice** Guided Learning	**Teacher Edition** Learn **Technology** Digi **Student Book** Guided Learning Pages Hands-on Activity	Students-already in pairs /small, homogenous ability groups; Teacher circulates between groups; Teacher, anecdotally, captures student thinking **Small Group w/Teacher circulating among groups** Revisit Concrete and Model Drawing; Reteach Teacher spends majority of time with struggling learners; some time with on level, and less time with advanced groups Games and Activities can be done at this time

INDEPENDENT PRACTICE	**Independent Practice** *A formal formative assessment*	**Teacher Edition** Let's Practice **Student Book** Let's Practice **Differentiation Options** All: Workbook Extra Support: Reteach On Level: Extra Practice Advanced: Enrichment	**Let's Practice** determines readiness for Workbook and small group work and is used as formative assessment; Students not ready for the Workbook will use Reteach. The Workbook is continued as Independent Practice. Manipulatives **CAN** be used as a communications tool as needed. Completely Independent On level/advance learners should finish all workbook pages.
ADDITIONAL PRACTICE	**Extending the Lesson**	Math Journal Problem of the Lesson Interactivities Games	
	Lesson Wrap Up	Problem of the Lesson Homework (Workbook , Reteach, or Extra Practice)	Workbook or Extra Practice Homework is only assigned when students fully understand the concepts (as additional practice) Reteach Homework (issued to struggling learners) should be checked the next day
POST TEST	**End of Chapter Wrap Up and Post Test**	**Teacher Edition** Chapter Review/Test Put on Your Thinking Cap **Student Workbook** Put on Your Thinking Cap **Assessment Book** Test Prep	Use Chapter Review/Test as "review" for the End of Chapter Test Prep. Put on your Thinking Cap prepares students for novel questions on the Test Prep; Test Prep is graded/scored. The Chapter Review/Test can be completed • Individually (e.g. for homework) then reviewed in class • As a 'mock test' done in class and doesn't count • As a formal, in class review where teacher walks students through the questions Test Prep is completely independent; scored/graded Put on Your Thinking Cap (green border) serve as a capstone problem and are done just before the Test Prep and should be treated as Direct Engagement. By February, students should be doing the Put on Your Thinking Cap problems on their own.

Math Background

During their elementary mathematics education, students were exposed to the following:

- Represent numbers up to 10,000 in word form, standard form, and expanded form
- Recognize the value of each digit in a number
- Estimation to check the reasonableness of sums and differences
- Multiplication tables up to 10 x 10
- Divide a 2 digit quotient by 1 digit divisor

In this unit, the students extend their learning to the following:

- Place value concepts extended to 6-digit numbers
- Power of ten
- Compare and order multi-digit whole numbers
- Add and subtract whole numbers with regrouping
- Various estimation strategies
- Factors
- Multiples, Least Common Multiple, Greatest Common Multiple
- Double Division Method
- Multiplication/Division Strategy using place value concepts
- Solve 3-step real-world problems including 4 operations

Throughout the unit, students review and extend place value concepts, addition and subtraction algorithms, estimation, and basic understanding of multiplication/division. In this unit, students are introduced to various new concepts/models. The chip-trading model is introduced in chapter 1 to familiarize students with the power of ten to assist with the beginning understanding of multiplication and division. Each color chip represents a specific power of ten. Students will continue to use reasonable estimation strategies, finding which method of estimation works easiest in their heads. Finally, students' use of multiplication will extend into their understanding of using the standard algorithm to multiply/divide 4 digits by 1 digit, and 2 digits by 2 digits. The students' fluent understanding of basic multiplication facts will assist in finding factors (GCF), multiples (LCM) and prime/composite numbers.

Potential Student Misconceptions

Chapter 1:

- When given the word form, students omit zeros when writing standard or expanded form.
- Students may not align the numbers by place value, but comparing only first digit. Use place value chart.
- Students lose track of how many units are in a place value column when it has been regrouped twice.

Chapter 2:

- Students may not know which place to round to when they do not have the same number of digits.
- Students may only name factors they are familiar with.
- Students confuse factors and multiples.
- Students may forget to write the zero in the ones place when multiplying the tens.

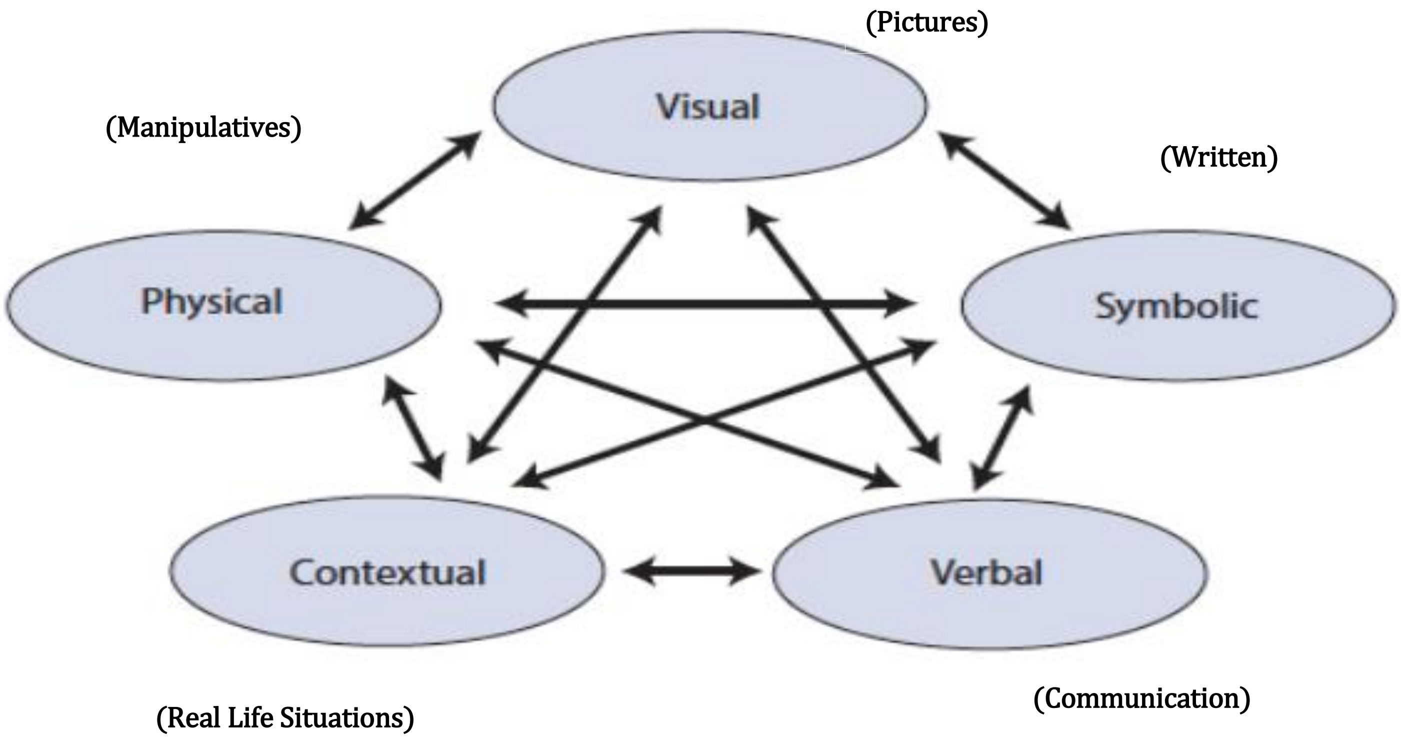
Chapter 3:

- Students may forget to add the regrouped numbers when they multiply each place.
- When multiplying with zeros, students multiply the regrouped number in the place where the zero is, instead of adding the regrouped number.
- Students may place the first digit of the quotient in the wrong place.
- Some students may ignore zeros in the dividend when dividing.

PARCC Assessment Evidence/Clarification Statements

CCSS	Evidence Statement	Clarification	Math Practices
4.OA.1-1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.	i) Tasks have "thin context" 1 or no context.	MP.2, MP.4
4.OA.1-2	Represent verbal statements of multiplicative comparisons as multiplication equations	i) Tasks have "thin context" or no context	MP.2, MP.4
4.OA.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	i) See the OA Progression document, especially p. 29 and Table 2, Common Multiplication and Division situations on page 89 of CCSSM. ii) Tasks sample equally the situations in the third row of Table 2 on page 89 of CCSSM.	MP.1, MP.4, MP.5
4.OA.3-2	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, in which remainders must be interpreted.	i) Assessing reasonableness of answer is not assessed here. ii) Tasks involve interpreting remainders. iii) Multi-step problems must have at least 3 steps. iv) See p. 30 of the OA Progression document.	MP.1, MP.2, MP.4, MP.7
4.NBT.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.		MP.7
4.NBT.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	i) Tasks assess conceptual understanding, e.g. by including a mixture of expanded form, number names, and base ten numerals within items.	MP.7
4.NBT.5-1	Multiply a whole number of up to four digits by a one-digit whole number using strategies based on place value and the properties of operations.	i) Tasks do not have a context.	MP.7
4.NBT.6-1	Find whole-number quotients and remainders with up to three-digit dividends and one digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.	i) Tasks do not have a context. ii) Tasks may include remainders of 0 in no more than 20% of the tasks.	MP.7, MP.8
4.NBT.Int.1	Perform computations by applying conceptual understanding of place value, rather than by applying multi-digit algorithms.	i) Tasks do not have a context.	MP.1, MP.7

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

> **Concrete:** “Doing Stage”: Physical manipulation of objects to solve math problems.
>
> **Pictorial:** “Seeing Stage”: Use of imaged to represent objects when solving math problems.
>
> **Abstract:** “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

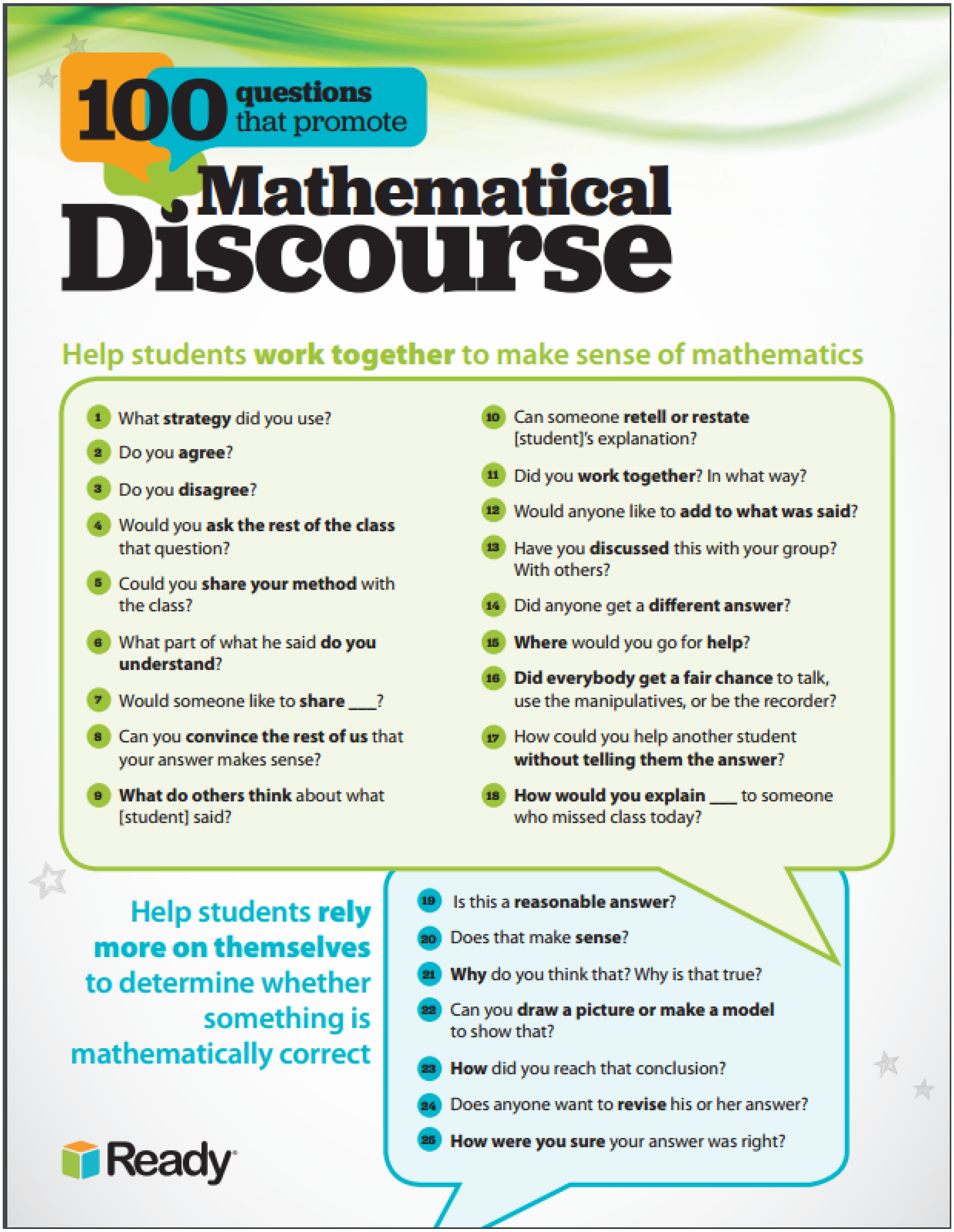
While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.



100 questions that promote Mathematical Discourse

Help students **work together** to make sense of mathematics

1. What **strategy** did you use?
2. Do you **agree**?
3. Do you **disagree**?
4. Would you **ask the rest of the class** that question?
5. Could you **share your method** with the class?
6. What part of what he said **do you understand**?
7. Would someone like to **share** ___?
8. Can you **convince the rest of us** that your answer makes sense?
9. **What do others think** about what [student] said?
10. Can someone **retell or restate** [student]'s explanation?
11. Did you **work together**? In what way?
12. Would anyone like to **add to what was said**?
13. Have you **discussed** this with your group? With others?
14. Did anyone get a **different answer**?
15. **Where** would you go for **help**?
16. **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
17. How could you help another student **without telling them the answer**?
18. **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

19. Is this a **reasonable answer**?
20. Does that make **sense**?
21. **Why** do you think that? Why is that true?
22. Can you **draw a picture or make a model** to show that?
23. **How** did you reach that conclusion?
24. Does anyone want to **revise** his or her answer?
25. **How were you sure** your answer was right?

Ready

Help students learn to **reason mathematically**

26 **How did you begin** to think about this problem?

27 What is **another way** you could solve this problem?

28 How could you **prove** ______?

29 Can you **explain how your answer is different from or the same as** [student]'s answer?

30 Let's **break the problem into parts**. What would the parts be?

31 Can you **explain this part more specifically**?

32 Does that **always work**?

33 Can you think of a case where that **wouldn't work**?

34 How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

35 What do you need to do **next**?

36 What have you **accomplished**?

37 What are your **strengths and weaknesses**?

38 Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39 What is this problem about? What can you **tell me about it**?

40 Do you need to **define or set limits** for the problem?

41 How would you **interpret** that?

42 Could you **reword that in simpler terms**?

43 Is there something that can be **eliminated** or that is **missing**?

44 Could you **explain** what the problem is asking?

45 What **assumptions** do you have to make?

46 What do you **know** about this part?

47 Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

Help students learn to **connect mathematics, its ideas, and its application**

74 What is the **relationship** between ___ and ___?

75 Have we ever solved a problem **like this before**?

76 What uses of mathematics did you find in the **newspaper** last night?

77 What is the **same**?

78 What is **different**?

79 Did you use skills or build on concepts that were **not necessarily mathematical**?

80 Which **skills or concepts** did you use?

81 What **ideas** have we explored before that were useful in solving this problem?

82 Is there a **pattern**?

83 **Where else** would this strategy be useful?

84 How does this **relate** to ___?

85 Is there a **general rule**?

86 Is there a **real-life situation** where this could be used?

87 How would your method work with **other problems**?

88 What other problem does this seem to **lead to**?

Help students **persevere**

89 Have you tried making a **guess**?

90 **What else** have you tried?

91 Would **another method** work as well or better?

92 Is there **another way** to draw, explain, or say that?

93 Give me another **related problem**. Is there an easier problem?

94 How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95 What was **one thing you learned** (or two, or more)?

96 Did you **notice any patterns**? If so, describe them.

97 What **mathematics topics** were used in this investigation?

98 What were the **mathematical ideas** in this problem?

99 What is mathematically **different about these two situations**?

100 What are the **variables** in this problem? What stays **constant**?

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students’ conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

3-5 Math Fact Fluency Expectation

3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)

3.NBT.A.2: Add/subtract within 1000

4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm

5.NBT.B.5: Multi-digit multiplication/ Use of Standard Algorithm

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

Assessment Framework

Unit 1 Assessment / Authentic Assessment Recommended Framework			
Assessment	**CCSS**	**Estimated Time**	**Format**
Diagnostic Assessment (IREADY)		1-2 blocks	Individual
Chapter 1			
Portfolio/Authentic Assessment #1	3.NBT.2	30 mins	Individual
Optional Chapter Test 1/ Performance Task	4.NBT.1-2, 4.NBT.4, 4.OA.5	1 block	Individual
Chapter 2			
Portfolio/Authentic Assessment #2	4.NBT.4	30 mins	Individual
Optional Chapter Test 2 Performance Task	4.OA.3-4, 4.NBT.2, 4.NBT.5	1 block	Individual
Chapter 3			
Portfolio/Authentic Assessment #3	4.NBT.6	30 mins	Individual
Optional Chapter Test 3/ Performance Task	4.NBT.5-6, 4.OA.3	1 block	Individual
Eureka Math Module 1: Algorithms for Addition/ Subtraction (Topic E, F)			
Optional End of Module Assessment	4.OA.3 4.NBT.4	1 Block	Individual
Portfolio/Authentic Assessment #4	4.NBT.1, 4.OA.1, 4.NBT.5	30 mins	Individual
Grade 4 Interim Assessment 1 (IREADY)	4.NBT.4-6	1-2 Block	Individual

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

Connections to the Mathematical Practices

1	**Make sense of problems and persevere in solving them** Mathematically proficient students in grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2	**Reason abstractly and quantitatively** Mathematically proficient fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3	**Construct viable arguments and critique the reasoning of others** In fourth grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.

4	**Model with mathematics**
	Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
5	**Use appropriate tools strategically**
	Mathematically proficient fourth graders consider the available tools(including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
6	**Attend to precision**
	As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
7	**Look for and make use of structure**
	In fourth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
8	**Look for and express regularity in repeated reasoning**
	Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

Authentic Assessment #1 – Place Value Addition

Name:____

The number sentence below can be solved using tens and ones.

67 + 25 = ___?_____ tens and _____?___ ones

Select **2** sets of numbers from each column to make the number sentence true.

Show all work and explain in writing how you arrived at your answer.

Tens	Ones
2	**2**
6	**5**
8	**10**
9	**12**

Place Value Addition

3.NBT.2: Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (A range of algorithms may be used.)

Mathematical Practice: 1

SOLUTION:

Tens	Ones
2	**2**
6	**5**
8	**10**
9	**12**

8 from the tens column must be paired with the 12 from the ones column.
and
9 from the tens column must be paired with the 2 from the ones column.

Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
Student pairs 2 correct combinations from each column. Clearly constructs and communicates a complete response based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction relationship Response includes an **efficient** and logical progression of steps.	**Student pairs 2 correct combinations from each column.** Clearly constructs and communicates a complete response based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction Response includes a logical progression of steps	**Student pairs one pair of correct numbers from each column.** Constructs and communicates a complete response based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction Response includes a logical but incomplete progression of steps. Minor calculation errors	**Student can only select one correct number.** Constructs and communicates an incomplete response based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction Response includes an incomplete or Illogical progression of steps.	**Student cannot select one correct number.** The student shows no work or justification.

4th Grade Authentic Assessment #2 – Zoo Animals

Name:_______________________________________

The animals at the zoo were weighed.

Name 3 animal pairs that have a difference in weight that is greater than 2,000.

Show all work including the actual differences for the 3 pairs of animals.

Animal	Weight in Pounds
Giraffe	2,685
Polar Bear	685
Hippopotamus	3,086
Cheetah	144
Buffalo	1600

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Mathematical Practice: 1, 3, 6

Type: Individual

SOLUTION: **Pair 1: Giraffe & Cheetah (2,541 pounds)** **Pair 2: Hippopotamus & Cheetah (2,942 pounds)** **Pair 3: Hippopotamus & Polar Bear (2,401 pounds)**				
Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Accurately and **quickly** adds and subtracts multi-digit whole numbers using the standard algorithm. Response includes an **efficient** and logical progression of steps.	Accurately and **in a timely manner** adds or subtracts multi-digit whole numbers using the standard algorithm. Response includes a **logical** progression of steps	**Accurately** adds and subtracts multi-digit whole numbers using the standard algorithm. Response includes a **logical but incomplete** progression of steps. Minor calculation errors.	Adds and subtracts multi-digit whole numbers using the standard algorithm with some level of accuracy. Response includes an **incomplete or Illogical** progression of steps.	Does not address task, unresponsive, unrelated or inappropriate.

4th Grade Authentic Assessment #3 – Mental Division Strategy

Name:____

Jillian says

I know that 20 times 7 is 140 and if I take away 2 sevens that leaves 126. So 126 ÷ 7 = 18.

- ***a.*** Is Jillian's calculation correct? Explain.
- b. Draw a picture showing Jillian's reasoning.
- c. Use Jillian's method to find 222 ÷ 6.

Minutes and Days

CCSS.MATH.CONTENT.4.NBT.6

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Mathematical Practices: 3, 4, and 6

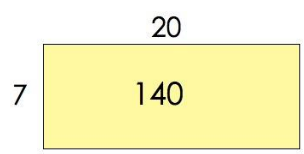
SOLUTION:

a. Jillian's reasoning is correct. She has found $20\times7=140$ and$2\times7=14$. This means that

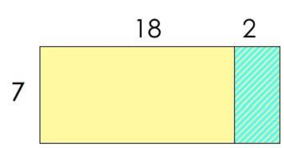
$$18\times7=(20-2)\times7=(20\times7)-(2\times7)=140-14=126.$$

The second equality uses the distributive property. These equations tell us that 126 ÷ 7 = 18.

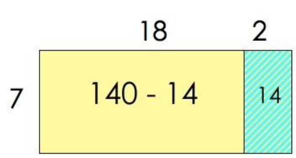
b. Jillian's initial idea of dividing 140 by 7 is represented here:



From there, Jillian decomposes the 20 sevens into 18 sevens and 2 sevens:



Lastly, Jillian recognizes that if the area of both rectangles combined would be 140, then she must subtract off the 2 extra sevens she used to get 140:



c. We have $40\times6=240$ and $3\times6=18$. So

$$37\times6=(40-3)\times6=(40\times6)-(3\times6)=240-18=222.$$

The second line uses the distributive property of multiplication.				
Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Student correctly answers and clearly constructs and communicates a complete response based on explanations/reasoning using : • Strategies based on place value, the properties of operations, and/or the relationship between multiplication and division Response includes an **efficient** and logical progression of steps.	Student correctly answers, clearly constructs, and communicates a complete response containing one minor calculation error based on explanations/reasoning using: • Strategies based on place value, the properties of operations, and/or the relationship between multiplication and division Response includes a **logical** progression of steps	Student correctly answers only two parts, clearly constructs, and communicates a response containing calculation and/or conceptual errors based on explanations/reasoning using: • Strategies based on place value, the properties of operations, and/or the relationship between multiplication and division Response includes a **logical but incomplete** progression of steps. Minor calculation errors.	Student correctly answers only one part, clearly constructs, and communicates a response containing major calculation and/or conceptual errors based on explanations/reasoning using: • Strategies based on place value, the properties of operations, and/or the relationship between multiplication and division Response includes an **incomplete or Illogical** progression of steps.	The student shows no work or justification.

4^{th} Grade Authentic Assessment #4 – Thousands and Millions of Fourth Graders

Name:_______________________________________

There are almost 40 thousand fourth graders in Mississippi and almost 400 thousand fourth graders in Texas. There are almost 4 million fourth graders in the United States.

We write 4 million as 4,000,000. How many times more fourth graders are there in Texas than in Mississippi? How many times more fourth graders are there in the United States than in Texas? Use the approximate populations listed above to solve.

There are about 4 thousand fourth graders in Washington, D.C. How many times more fourth graders are there in the United States than in Washington, D.C.?

Performance Task Scoring Rubric:

Thousands and Millions of Fourth Graders

CCSS.MATH.CONTENT.4.NBT.1

Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700÷70=10 by applying concepts of place value and division.

CCSS.MATH.CONTENT.4.OA.1

Interpret a multiplication equation as a comparison, e.g., interpret 35=5×7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

CCSS.MATH.CONTENT.4.NBT.5

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Mathematical Practices: 2, and 6

SOLUTION:

We write 4 thousand as 4,000
We write 40 thousand as 40,000
We write 400 thousand as 400,000

The value of each place is ten times the value of the place immediately to the right.

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	
		x10	4	0	0	0	Wash., D.C.
	x10	4	0	0	0	0	Mississippi
x10	4	0	0	0	0	0	Texas
4	0	0	0	0	0	0	United States

So:
40,000 is 10 times 4,000
400,000 is 10 times 40,000.

4,000,000 is 10 times 400,000.

Thus, 400,000 = 10 × 40,000, and there are about 10 times as many fourth graders in Texas as there are in Mississippi.
Also, 4,000,000 = 10 × 400,000, and there are about 10 times as many fourth graders in the US as there are in Texas.
Finally, to go from 4,000 to 4,000,000, we have to multiply by 10 three times. We see that

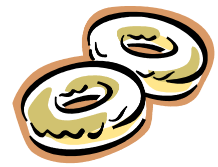
$$10\times10\times10=10\times100=1000$$

So there are about 1,000 times as many fourth graders in the US as there are in Washington DC.

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Student correctly answers and clearly constructs and communicates a complete response based on explanations/reasoning using : • Concepts of place value and division • Strategies based on place value and the properties of operations Response includes an **efficient** and logical progression of steps.	Student correctly answers and clearly constructs and communicates a complete response with one minor calculation error based on explanations/reasoning using: • Concepts of place value and division • Strategies based on place value and the properties of operations. Response includes a **logical** progression of steps	Student answers, clearly constructs, and communicates a complete response with minor calculation errors based on explanations/reasoning using: • Concepts of place value and division • Strategies based on place value and the properties of operations Response includes a **logical but incomplete** progression of steps. Minor calculation errors.	Student answers, clearly constructs, and communicates a complete response with major calculation errors and/or conceptual errors based on explanations/reasoning using: • Concepts of place value and division • Strategies based on place value and the properties of operations Response includes an **incomplete or Illogical** progression of steps.	The student shows no work or justification.

4.OA.1

Donut Shop



Show your thinking using pictures, numbers, or words.

1. At the party, Emile ate 2 donuts. Lucas ate four times as many donuts as Emile. How many donuts did they eat altogether?

2. Frank ordered 24 cookies from the donut shop. That is three times as many cookies as Jenny ordered and six times as many cookies as Barb ordered. How many cookies did they order altogether?

   How many more cookies did Frank order than Jenny and Barb together?

3. The donut shop made 28 chocolate donuts. That is 7 times as many strawberry donuts as they made. How many more chocolate donuts did the shop make than strawberry donuts?

4. Students in Mr. Juarez's class predicted the number of donuts they could eat. The chart below shows their predictions.

Rosa	Jeremy	Frank	Jill	Emile	Troy	Sandy	Lucas	Shante	Molly
2	18	4	6	3	14	9	12	36	8

Compare these numbers in as many ways as you can.

Example: *Shante said he could eat three times as many donuts as Lucas.*

4.OA. 2 Selling Candy

Sarah and Jose are both selling candy for a school fundraiser. Sarah's total amount of money is 7 times greater than the number of days that they have sold candy. Jose's total amount of money is $3 more than the number of days that they have sold candy. Complete the table below showing the amount of money that they have earned for all 7 days selling candy.

Sarah	
Days	Total
1	7
2	14
3	
4	
5	
6	
7	

Jose	
Days	Total
1	4
2	5
3	
4	
5	
6	
7	

Part 2:

For both Sarah and Jose calculate how much money would be made on the tenth day. Write an equation to explain your reasoning.

Part 3:

Write a sentence comparing how to calculate how much Sarah and Jose each made on the tenth day.

4.OA. 3
How Many Teams?

In eastern North Carolina there are 3,277 fourth graders signed up for basketball. In western North Carolina there are 2,981 fourth graders signed up for basketball. In the Piedmont region there are 1,512 players signed up. Every player will get placed on a team in their region of the state.

Part 1:
The league wants to place 9 players on each team? Leftover players will be added to teams, so some teams will have ten players. How many teams will have 9 players in each region of the state? How many teams will have 10 players in each region of the statewide? Statewide, how many teams have 9 players and how many teams have 10 players? Explain your reasoning.

Part 2:
In order to maximize playing time, the league decides to only place 7 players on each team. If there are extra players, some teams will have 8 players. How many teams will have 7 players in each region of the state? How many teams will have 8 players in each region of the state? Statewide, how many teams have 7 players and how many teams have 8 players? Explain your reasoning.

Factors and Multiples

A Ride On A Bus

Part 1:

Eighty fourth grade students at Andrews Elementary School are going on a field trip. Their teachers need to put between 3 and 25 students in each group to visit the shark tank. How many different ways can the teachers group their students so that each group has the same number of students?

Part 2:

If four groups of eight students ride bus 1, how many students will ride bus 2?

How many different ways can the teacher group the students on bus 2 so that each group has the same number of students? Explain your reasoning using pictures, numbers or words.

Arranging Chairs

Part 1:
There are 24 chairs in the art room. What are the different ways that the chairs can be arranged into equal groups if you want at least 2 groups and want at least 2 chairs in each group? How do you know that you have found every arrangement? Write division equations to show your answers.

Part 2:
There are 48 chairs in the multi-purpose room. What are the different ways that the chairs can be arranged into equal groups if you want at least 2 groups and want at least 2 chairs in each group? How do you know that you have found every arrangement? Write division equations to show your answers.

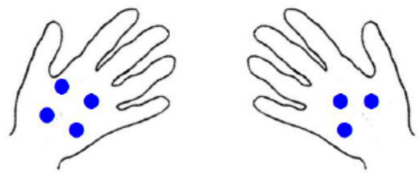
Part 3:
What relationship do you notice about the size of the groups if the chairs were arranged in 4 groups in both Part 1 and Part 2? What about if the chairs were arranged in 8 groups? Explain why you think this relationship exists.

Visual Vocabulary

Visual Definition

The terms below are for teacher reference only and are not to be memorized by students. *Teachers should first present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or use them with words, models, pictures, or numbers.*

Chapter 1

compare	 **4 is more than 3.**	To decide if one number is greater than, less than, or equal to.
digit	**0 1 2 3 4** **5 6 7 8 9**	Any of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. (also known as base-ten numerals)
standard form	**354,973**	A number written with one digit for each place value.
word form	**The word form of 234 is two hundred, thirty-four.**	A way of using words to write a number.
expanded form	**347.392 =** 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000)	A way to write numbers that shows the place value of each digit.

place value

MILLIONS			THOUSANDS			ONES		
hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
7	4	5,	3	0	9,	2	8	1

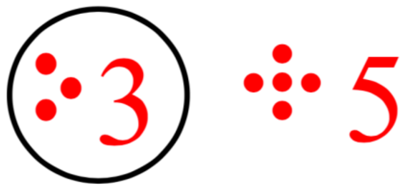
The value of the place of a digit in a number.

greater than

$5 > 3$

Greater than is used to compare two numbers when the first number is larger than the second number.

less than



$3 < 5$

Less than is used to compare two numbers when the first number is smaller than the second number.

Chapter 2

estimate

Close to 1 Close to 1

$\frac{3}{4} + \frac{5}{6} \approx 2$

is approximately equal to

A number close to an exact amount, an estimate tells *about* how much.

product

$5 \times 3 = 15$

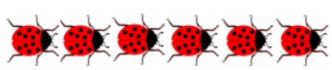
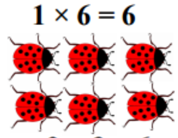
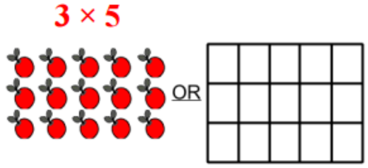
The answer to a multiplication problem.

factor

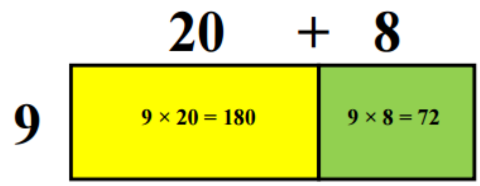
$2 \times 6 = 12$

factors

The whole numbers that are multiplied to get a product.

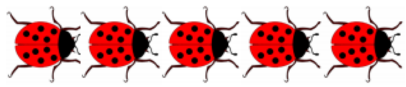
quotient	quotient $\begin{array}{r} 15\text{ r. }2 \\ 9\overline{)137} \end{array}$	The result of the division of one quantity by another.
divisor	$8\overline{)578}$ divisor	The quantity by which another quantity is to be divided.
multiple	12 is a multiple of 3 (and of 4) because 3 x 4 = 12	A product of a given whole number and any other whole number.
least common multiple	6, 12, 18, 24, 30, 36, 42… 8, 16, 24, 32, 40, 48, 56… LCM = 24	LCM. The smallest common multiple of a set of two or more numbers.
composite number	 $1 \times 6 = 6$  $2 \times 3 = 6$ 6 is a composite number.	A number greater than 0 that has more than two different factors.
array	3 rows of 5 3×5 	An arrangement of objects in equal rows.

area model



$9 \times 28 = (9 \times 20) + (9 \times 8) = 252$

prime number



$1 \times 5 = 5$

5 is a prime number.

A whole number greater than 0 that has exactly two different factors, 1 and itself.

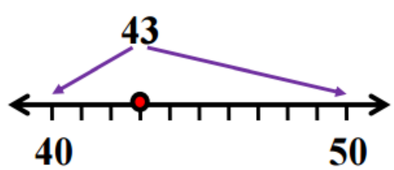
reasonableness

What is the product of 57 and 34?
A. 1,938 C. 5,738
B. 3,208 D. 8,698

Use estimation to eliminate unreasonable choices.
$60 \times 30 = 1{,}800$
B, C, and D are not close to 1,800.
The answer is A.

An answer that is based on good number sense.

round a whole number



To find the nearest ten, hundred, thousand, (and so on).

Chapter 3

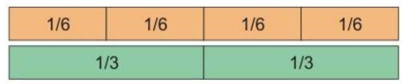
remainder

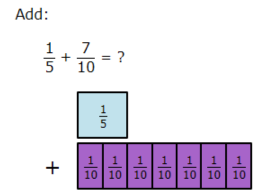
remainder

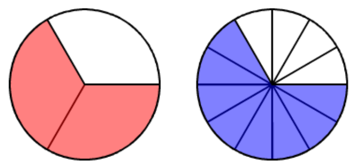
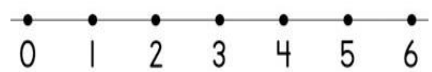
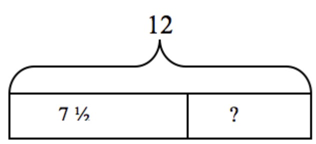
$$9\overline{)137}\quad 15 \text{ r. } 2$$

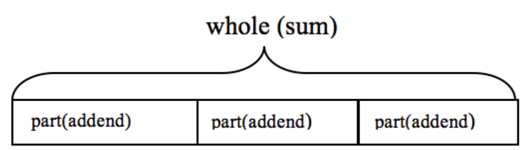
The number that is left over after a whole number is divided equally by another.

<u>Teaching Multiple Representations</u>

<table>
<tr><th colspan="2">Concrete and Pictorial Representations</th></tr>
<tr><td>Equal Partitioning and Unitizing
Using Visual Fraction Models
Fraction Strips
Fraction Circles
Number line</td><td>





</td></tr>
<tr><td>Bar Model
</td><td>Leticia read 7 ½ books for the read-a-thon. She wants to read 12 books in all. How many more books does she have to read?




12 - 7 ½ = ? or 7 ½ + ? = 12 so Leticia needs to read 4 ½ more books.</td></tr>
<tr><td>Equivalent Fractions</td><td>For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)</td></tr>
<tr><td>Benchmark Fractions</td><td>1/2, 1/3, 1/4, 1/5, 1/6. 1/8, 1/10</td></tr>
<tr><th colspan="2">Abstract Representations</th></tr>
<tr><td>Basic Mathematical Properties</td><td>Additive Inverse

Example: 7 + (-7) = 0</td></tr>
</table>

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

Websites

Illustrative Math: http://illustrativemathematics.org/
PARCC: http://www.parcconline.org/samples/item-task-prototypes
NJDOE: http://www.state.nj.us/education/modelcurriculum/math/ (username: model; password: curriculum)
DANA: http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html
New York:http://www.p12.nysed.gov/assessment/common-core-sample-questions/
Delaware: http://www.doe.k12.de.us/assessment/CCSS-comparison-docs.shtml

Extensions and Sources

Think Central
https://www-k6.thinkcentral.com/ePC/start.do

Common Core Tools
http://commoncoretools.me/
http://www.ccsstoolbox.com/
http://www.achievethecore.org/steal-these-tools

Achieve the Core
http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

Manipulatives

http://nlvm.usu.edu/en/nav/vlibrary.html
http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000

Problem Solving Resources

IllustrativeMath Project
http://illustrativemathematics.org/standards/k8

The site contains sets of tasks that illustrate the expectations of various CCSS in grades K–8 grade and high school. More tasks will be appearing over the coming weeks. Eventually the sets of tasks will include elaborated teaching tasks with detailed information about using them for instructional purposes, rubrics, and student work.

Inside Mathematics
http://www.insidemathematics.org/index.php/tools-for-teachers
Inside Mathematics showcases multiple ways for educators to begin to transform their teaching practices. On this site, educators can find materials and tasks developed by grade level and content area.

Engage NY
http://www.engageny.org/video-library?f[0]=im_field_subject%3A19

IXL
http://www.ixl.com/

Georgia Department of Education
https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx
Georgia State Educator have created common core aligned units of study to support schools as they implement the Common Core State Standards.